Basic Differentiation - A Refresher

v3. March 2003

Contents

Foreword

The material in this refresher course has been designed to enable you to cope better with your university mathematics programme. When your programme starts you will find that the ability to differentiate confidently will be invaluable. We think that this is so important that we are making this course available for you to work through either before you come to university, or during the early stages of your programme.

Preliminary work

You are advised to work through the companion booklet *An Algebra Refresher* before embarking upon this basic differentiation revision course.

How to use this booklet

You are advised to work through each section in this booklet in order. You may need to revise some topics by looking at an AS-level textbook which contains information about differentiation.

You should attempt a range of questions from each section, and check your answers with those at the back of the booklet. The more questions that you attempt, the more familiar you will become with these vital topics. We have left sufficient space in the booklet so that you can do any necessary working within it. So, treat this as a work-book.

If you get questions wrong you should revise the material and try again until you are getting the majority of questions correct.

If you cannot sort out your difficulties, do not worry about this. Your university will make provision to help you with your problems. This may take the form of special revision lectures, self-study revision material or a drop-in mathematics support centre.

Reminders

Use this page to note topics and questions which you found difficult.

Seek help with these from your tutor or from other university support services as soon as possible.

Introduction

Differentiation is an aspect of calculus that enables us to determine how one quantity changes with regard to another.

In this booklet we will not however be concerned with the applications of differentiation, or the theory behind its development, but solely with the mechanisms by which the process of differentiation may be quickly and effectively carried out.

Each section that follows will introduce a particular technique followed by several problems for you to practice what you have learned.

At the end of the booklet you will find answers for each of the sections.

Included are some pages for you to make notes that may serve as a reminder to you of any possible areas of difficulty. You should seek help with such areas of difficulty from your tutor or other university support services.

1. Differentiation of a simple power

To differentiate $s = t^n$. • Bring the existing power down and use it to multiply. • Reduce the old power by one and use this as the new power. **Answer** $\dfrac{ds}{dt} = nt^{n-1}$	Example $s = t^3$ $\dfrac{ds}{dt} = 3t^{3-1}$ $= 3t^2$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $s = t^9$, $\dfrac{ds}{dt}$	(b) $v = t^4$, $\dfrac{dv}{dt}$	(c) $a = t^5$, $\dfrac{da}{dt}$	(d) $y = x^7$, $\dfrac{dy}{dx}$
(e) $y = x^{12}$, $\dfrac{dy}{dx}$	(f) $V = r^3$, $\dfrac{dV}{dr}$	(g) $P = v^6$, $\dfrac{dP}{dv}$	(h) $A = r^2$, $\dfrac{dA}{dr}$
(i) $T = \theta^8$, $\dfrac{dT}{d\theta}$	(j) $R = t^{19}$, $\dfrac{dR}{dt}$	(k) $v = t^{11}$, $\dfrac{dv}{dt}$	(l) $p = V^{10}$, $\dfrac{dp}{dV}$

2. Differentiation of a unit power

To differentiate $s = t$.

- The phrase 'a unit power' refers to the fact that the power is 1.
- Note that t is the same as t^1. We can use the previous rule.
- Differentiating the term $t = t^1$ gives $1t^0 = 1$

Answer $\dfrac{ds}{dt} = 1$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a)	$r = t,$	$\dfrac{dr}{dt}$	(b)	$v = t,$	$\dfrac{dv}{dt}$	(c)	$a = t,$	$\dfrac{da}{dt}$	(d)	$y = x,$	$\dfrac{dy}{dx}$
(e)	$z = y,$	$\dfrac{dz}{dy}$	(f)	$V = r,$	$\dfrac{dV}{dr}$	(g)	$P = v,$	$\dfrac{dP}{dv}$	(h)	$W = z,$	$\dfrac{dW}{dz}$
(i)	$T = \theta,$	$\dfrac{dT}{d\theta}$	(j)	$Q = r,$	$\dfrac{dQ}{dr}$	(k)	$m = s,$	$\dfrac{dm}{ds}$	(l)	$x = y,$	$\dfrac{dx}{dy}$

3. Differentiation of a constant

To differentiate $s = a$ where a is a constant. • An isolated constant always differentiates to zero. **Answer** $\dfrac{ds}{dt} = 0$	Example $s = 4$ $\dfrac{ds}{dt} = 0$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a)	$s = 5,$	$\dfrac{ds}{dt}$	(b)	$v = 0.6,$	$\dfrac{dv}{dt}$	(c)	$\theta = \dfrac{1}{2},$	$\dfrac{d\theta}{dt}$	(d)	$r = 1,$	$\dfrac{dr}{d\theta}$
(e)	$T = \pi,$	$\dfrac{dT}{dt}$	(f)	$z = 29.6,$	$\dfrac{dz}{dx}$	(g)	$x = 3^{-1},$	$\dfrac{dx}{dt}$	(h)	$f = 2.7,$	$\dfrac{df}{dx}$
(i)	$p = 4,$	$\dfrac{dp}{dr}$	(j)	$q = \sqrt{2},$	$\dfrac{dq}{dt}$	(k)	$w = \dfrac{3}{4},$	$\dfrac{dw}{dz}$	(l)	$m = 2.7,$	$\dfrac{dm}{d\theta}$

4. Differentiation of a simple power multiplied by a constant

To differentiate $s = at^n$ where a is a constant. • Bring the existing power down and use it to multiply. • Reduce the old power by one and use this as the new power. **Answer** $\dfrac{ds}{dt} = ant^{n-1}$	Example $s = 3t^4$ $\dfrac{ds}{dt} = 4 \times 3t^{4-1}$ $= 12t^3$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $s = 3t^4$, $\dfrac{ds}{dt}$	(b) $y = 7x^3$, $\dfrac{dy}{dx}$	(c) $r = 0.4\theta^5$, $\dfrac{dr}{d\theta}$	(d) $v = 4r^3$, $\dfrac{dv}{dr}$
(e) $A = \pi r^2$, $\dfrac{dA}{dr}$	(f) $T = 6\theta^4$, $\dfrac{dT}{d\theta}$	(g) $v = 5t^9$, $\dfrac{dv}{dt}$	(h) $p = 6v^3$, $\dfrac{dp}{dv}$
(i) $y = 14x^2$, $\dfrac{dy}{dx}$	(j) $r = 5t^7$, $\dfrac{dr}{dt}$	(k) $P = 7v^6$, $\dfrac{dP}{dv}$	(l) $C = 12N^8$, $\dfrac{dC}{dN}$

Notes Page

5. Differentiation of a unit power multiplied by a constant

To differentiate $s = at$ where a is a constant.	Example
• The result is always the same as the constant.	$s = 3t$
Answer $\dfrac{ds}{dt} = a$	$\dfrac{ds}{dt} = 3$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $y = 5x$, $\dfrac{dy}{dx}$	(b) $s = 6t$, $\dfrac{ds}{dt}$	(c) $p = 4v$, $\dfrac{dp}{dv}$
(d) $C = 7n$, $\dfrac{dC}{dn}$	(e) $Q = 0.5v$, $\dfrac{dQ}{dv}$	(f) $z = \pi w$, $\dfrac{dz}{dw}$
(g) $N = 1.7n$, $\dfrac{dN}{dn}$	(h) $\theta = \dfrac{1}{3}t$, $\dfrac{d\theta}{dt}$	(i) $r = \dfrac{1}{4}t$, $\dfrac{dr}{dt}$
(j) $s = 3.14\omega$, $\dfrac{ds}{d\omega}$	(k) $w = 5z$, $\dfrac{dw}{dz}$	(l) $R = 9.6\phi$, $\dfrac{dR}{d\phi}$

6. Differentiation of a general power

To differentiate $s = t^n$ for any value of n.	Example
• Bring the existing power down and use it to multiply.	$s = t^{-3.4}$
• Reduce the old power by one and use this as the new power.	$\frac{ds}{dt} = -3.4t^{-3.4-1}$
Answer $\frac{ds}{dt} = nt^{n-1}$	$= -3.4t^{-4.4}$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $s = t^{2.8}$, $\frac{ds}{dt}$	(b) $y = x^{2/3}$, $\frac{dy}{dx}$	(c) $r = 0.4\theta^{1/4}$, $\frac{dr}{d\theta}$
(d) $v = 4r^{-3}$, $\frac{dv}{dr}$	(e) $A = r^{1/2}$, $\frac{dA}{dr}$	(f) $T = \theta^{-6}$, $\frac{dT}{d\theta}$
(g) $v = t^{-1}$, $\frac{dv}{dt}$	(h) $p = v^{-3.5}$, $\frac{dp}{dv}$	(i) $y = x^{3/4}$, $\frac{dy}{dx}$
(j) $r = t^{-1/3}$, $\frac{dr}{dt}$	(k) $P = v^{1.8}$, $\frac{dP}{dv}$	(l) $C = N^{-6}$, $\frac{dC}{dN}$

7. Differentiation of a general power multiplied by a constant

To differentiate $s = at^n$ for any value of n where a is a constant.	Example
• Bring the existing power down and use it to multiply.	$s = 2.5t^{-4}$
• Reduce the old power by one and use this as the new power.	$\dfrac{ds}{dt} = -4 \times 2.5t^{-4-1}$
Answer $\dfrac{ds}{dt} = ant^{n-1}$	$= -10t^{-5}$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $y = 5x^{1.5}$, $\dfrac{dy}{dx}$	(b) $s = 4t^{-3}$, $\dfrac{ds}{dt}$	(c) $p = 8v^{1/2}$, $\dfrac{dp}{dv}$
(d) $C = 6n^{-1}$, $\dfrac{dC}{dn}$	(e) $Q = 2v^{0.5}$, $\dfrac{dQ}{dv}$	(f) $z = 6w^{-2.5}$, $\dfrac{dz}{dw}$
(g) $N = 5.2n^{10}$, $\dfrac{dN}{dn}$	(h) $\theta = \dfrac{1}{5}t^6$, $\dfrac{d\theta}{dt}$	(i) $r = -5t^{-2}$, $\dfrac{dr}{dt}$
(j) $s = 2.5p^{2.5}$, $\dfrac{ds}{dp}$	(k) $w = 15z^{-0.1}$, $\dfrac{dw}{dz}$	(l) $v = -3r^{-4}$, $\dfrac{dv}{dr}$

8. Differentiation of a sum or difference of terms

To differentiate $s = f(t) + g(t)$ or $s = f(t) - g(t)$.	Example
• Differentiate $f(t)$ and $g(t)$ using previous rules.	$s = 3t^2 \pm 6t^{\frac{1}{3}}$
• Add or subtract the results.	$\dfrac{ds}{dt} = 2 \times 3t^{2-1} \pm \dfrac{1}{3} \times 6t^{\frac{1}{3}-1}$
Answer $\dfrac{ds}{dt} = \dfrac{df}{dt} + \dfrac{dg}{dt}$ or $\dfrac{ds}{dt} = \dfrac{df}{dt} - \dfrac{dg}{dt}$	$= 6t \pm 2t^{-\frac{2}{3}}$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a)	$s = t^2 + t^3$,	$\dfrac{ds}{dt}$	(b)	$v = 2t + 3t^4$,	$\dfrac{dv}{dt}$	(c)	$a = 3 + 4t$,	$\dfrac{da}{dt}$
(d)	$v = 5t - 10t^2$,	$\dfrac{dv}{dt}$	(e)	$a = 3t^{4.5} - 4t^{-1}$	$\dfrac{da}{dt}$	(f)	$s = 4t^{1/2} - 2.5t^{-2}$,	$\dfrac{ds}{dt}$
(g)	$y = x^2 + 3x^4$	$\dfrac{dy}{dx}$	(h)	$y = 2x^{-1} - x^{-2}$,	$\dfrac{dy}{dx}$	(i)	$x = 3y + \dfrac{1}{4}y^2$,	$\dfrac{dx}{dy}$

Notes Page

9. Differentiation of a simple fraction

To differentiate $s = \dfrac{a}{t^n}$ where a is a constant.	Example
• Rewrite as $s = at^{-n}$.	$s = \dfrac{2}{t^3}$ $= 2t^{-3}$
• Differentiate using previous rules.	$\dfrac{ds}{dt} = -3 \times 2t^{-3-1}$
Answer $\dfrac{ds}{dt} = -ant^{-n-1}$	$= -6t^{-4}$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $s = \dfrac{1}{t}$, $\dfrac{ds}{dt}$	(b) $s = \dfrac{2}{t^2}$, $\dfrac{ds}{dt}$	(c) $y = \dfrac{5}{x^3}$, $\dfrac{dy}{dx}$
(d) $p = \dfrac{2.4}{r^7}$, $\dfrac{dp}{dr}$	(e) $v = \dfrac{4.5}{t^{1/2}}$, $\dfrac{dv}{dt}$	(f) $a = \dfrac{10}{t^6}$, $\dfrac{da}{dt}$
(g) $v = \dfrac{4.9}{x^4}$, $\dfrac{dv}{dx}$	(h) $s = \dfrac{2.5}{\theta^3}$, $\dfrac{ds}{d\theta}$	(i) $x = \dfrac{\pi}{\theta}$, $\dfrac{dx}{d\theta}$
(j) $\theta = \dfrac{\sqrt{2}}{t^5}$, $\dfrac{d\theta}{dt}$	(k) $z = \dfrac{7}{2x}$, $\dfrac{dz}{dx}$	(l) $q = \dfrac{3}{r^{2.5}}$, $\dfrac{dq}{dr}$

10. Differentiation of fractions reducible to simpler fractions

To differentiate $s = \dfrac{a + bt^n}{ct^m}$.

- Rewrite as $s = \dfrac{a}{ct^m} + \dfrac{bt^n}{ct^m}$.
- Use power laws to simplify.
- Differentiate using previous rules.

Answer $\dfrac{ds}{dt} = -\dfrac{amt^{-m-1}}{c} + \dfrac{b}{c}(n-m)t^{n-m-1}$

Example

$$s = \frac{4 + 3t^2}{5t^3}$$

$$= \frac{4}{5t^3} + \frac{3t^2}{5t^3}$$

$$= \frac{4t^{-3}}{5} + \frac{3t^{-1}}{5}$$

$$\frac{ds}{dt} = -\frac{3 \times 4t^{-3-1}}{5} + (-1) \times \frac{3t^{-1-1}}{5}$$

$$= -\frac{12t^{-4}}{5} - \frac{3t^{-2}}{5}$$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $s = \dfrac{4t+1}{t}$, $\dfrac{ds}{dt}$	(b) $s = \dfrac{5t^3+3t}{t}$, $\dfrac{ds}{dt}$	(c) $v = \dfrac{5t^2+3t+1}{t^3}$, $\dfrac{dv}{dt}$
(d) $y = \dfrac{7+6x+5x^2}{x^2}$, $\dfrac{dy}{dx}$	(e) $y = \dfrac{3+5x}{4x}$, $\dfrac{dy}{dx}$	(f) $z = \dfrac{7+2t}{3t}$, $\dfrac{dz}{dt}$
(g) $\theta = \dfrac{2+5t+2t^3}{t^2}$, $\dfrac{d\theta}{dt}$	(h) $w = \dfrac{8+6z^2}{3z^4}$, $\dfrac{dw}{dz}$	(i) $x = \dfrac{3y+2y^2}{4y}$, $\dfrac{dx}{dy}$

Notes Page

11. Differentiation of a simple bracket

To differentiate $s = (at+b)^n$.	Example
• Bring the existing power down and use it to multiply.	$s = (3t+6)^5$
• Reduce the old power by 1 and use this as the new power.	
• Differentiate the expression inside the bracket.	
• Multiply your results together.	$\dfrac{ds}{dt} = 5 \times (3t+6)^{5-1} \times 3$
Answer $\dfrac{ds}{dt} = an(at+b)^{n-1}$	$= 15(3t+6)^4$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a)	$s = (2t+3)^6$,	$\dfrac{ds}{dt}$	(b)	$v = (5+2t)^7$,	$\dfrac{dv}{dt}$	(c)	$\theta = (0.4t+1)^{2.5}$,	$\dfrac{d\theta}{dt}$
(d)	$z = (\pi x+2.4)^{10}$,	$\dfrac{dz}{dx}$	(e)	$a = (5t-3)^{-2}$,	$\dfrac{da}{dt}$	(f)	$s = (3-6t)^{-3}$,	$\dfrac{ds}{dt}$
(g)	$y = (4x-1)^{1/2}$,	$\dfrac{dy}{dx}$	(h)	$y = (2+0.5t)^3$,	$\dfrac{dy}{dt}$	(i)	$z = (4x-6)^{-0.5}$,	$\dfrac{dz}{dx}$

12. Differentiation of a general bracket

To differentiate $s = [\,f(t)\,]^n$.	Example
• Bring the existing power down and use it to multiply.	$s = (5t^3 - 3t^2 + 1)^7$
• Reduce the old power by 1 and use this as the new power.	
• Differentiate $f(t)$ to give $\dfrac{df}{dt}$.	
• Multiply your results together.	$\dfrac{ds}{dt} = 7\times(5t^3 - 3t^2 + 1)^{7-1}\times(3 \times 5t^{3-1} - 2 \times 3t^{2-1} + 0)$
• Simplify.	$= 7(5t^3 - 3t^2 + 1)^6(15t^2 - 6t)$
Answer $\dfrac{ds}{dt} = n[\,f(t)\,]^{n-1}.\dfrac{df}{dt}$	$= 21t(5t^3 - 3t^2 + 1)^6(5t - 2)$

Practice: In the space provided write down the requested derivative for each of the following expressions.

(a) $s = (3t^2 + 2t + 4)^5$	$\dfrac{ds}{dt}$	(b) $r = (4\theta^2 - 2\theta + 1)^{-2}$	$\dfrac{dr}{d\theta}$	(c) $v = (8x^2 + 5x)^2$	$\dfrac{dv}{dx}$
(d) $\theta = (3t + 1)^{-1}$	$\dfrac{d\theta}{dt}$	(e) $p = (5v^2 + 2)^{-2}$	$\dfrac{dp}{dt}$	(f) $y = (t^3 + t + 1)^2$	$\dfrac{dy}{dt}$
(g) $z = (5x^3 + 1)^{-4}$	$\dfrac{dz}{dx}$	(h) $s = (2t + 1)^4$	$\dfrac{ds}{dt}$	(i) $x = (3y^2 - 4y)^3$	$\dfrac{dx}{dy}$

Notes Page

13. Miscellaneous problems

1. In each of the following determine $\dfrac{ds}{dt}$.

$(a)\quad s = \dfrac{2}{t}$	$(b)\quad s = t^{1/2}$	$(c)\quad s = 4t^{-2}$
$(d)\quad s = \dfrac{1}{3}t$	$(e)\quad s = \dfrac{2t+1}{t}$	$(f)\quad s = \dfrac{t^3 - 3t + 4}{t^2}$
$(g)\quad s = (5t+1)^4$	$(h)\quad s = (3t+1)^{-1}$	$(i)\quad s = \dfrac{2}{3t+1}$
$(j)\quad s = 5t^2 + 3t + 6$	$(k)\quad s = 2t^{-3} + t^4$	$(l)\quad s = (2t + 3t^{-1})^{-5}$

2. In each of the following determine $\dfrac{dy}{dx}$.

$(a)\quad y = x^{1/2}$	$(b)\quad y = (x+1)^{1/2}$	$(c)\quad y = \dfrac{3}{x} + 1$
$(d)\quad y = 5x + \dfrac{1}{x}$	$(e)\quad y = 5x^2$	$(f)\quad y = 7(x^2+1)^5$
$(g)\quad y = (2x+1)^{1/3}$	$(h)\quad y = \dfrac{7+2x}{x}$	$(i)\quad y = \dfrac{5}{3x^2}$
$(j)\quad y = 4x^{-2} + 3x^{-1}$	$(k)\quad y = 24 - 2x^{-3/4}$	$(l)\quad y = (2x^3 + 3x^2 - 5x)^2$

3. In each of the following determine $\frac{dv}{dt}$.

(a) $v = 3t$ (b) $v = 0.7t^{-3} - 6t^3$ (c) $v = (5t-6)^{-0.5}$

(d) $v = \frac{4}{t} + 3t$ (e) $v = 6$ (f) $v = 2t^{-2} - 3t^{-1} + 4 + 5t$

(g) $v = (6t-2)^{-0.5}$ (h) $v = \frac{3}{t^2}$ (i) $v = 3t - \frac{1}{t^{0.1}}$

(j) $v = 8t^4 - 3t^{1/2}$ (k) $v = (5t^{-2} + 3t^2)^3$ (l) $v = \pi - \pi t$

Answers to Problems

Section 1

(a) $9t^8$	(b) $4t^3$	(c) $5t^4$	(d) $7x^6$
(e) $12x^{11}$	(f) $3r^2$	(g) $6v^5$	(h) $2r$
(i) $8\theta^7$	(j) $19t^{18}$	(k) $11t^{10}$	(l) $10V^9$

Section 2

(a) 1	(b) 1	(c) 1	(d) 1
(e) 1	(f) 1	(g) 1	(h) 1
(i) 1	(j) 1	(k) 1	(l) 1

Section 3

(a) 0	(b) 0	(c) 0	(d) 0
(e) 0	(f) 0	(g) 0	(h) 0
(i) 0	(j) 0	(k) 0	(l) 0

Section 4

(a) $12t^3$	(b) $21x^2$	(c) $2\theta^4$	(d) $12r^2$
(e) $2\pi r$	(f) $24\theta^3$	(g) $45t^8$	(h) $18v^2$
(i) $28x$	(j) $35t^6$	(k) $42v^5$	(l) $96N^7$

Section 5

(a) 5	(b) 6	(c) 4	(d) 7
(e) 0.5	(f) π	(g) 1.7	(h) 1/3
(i) 1/4	(j) 3.14	(k) 5	(l) 9.6

Section 6

(a) $2.8t^{1.8}$ (b) $\frac{2}{3}x^{-1/3}$ (c) $0.1\theta^{-3/4}$ (d) $-12r^{-4}$

(e) $\frac{1}{2}r^{-1/2}$ (f) $-6\theta^{-7}$ (g) $-t^{-2}$ (h) $-3.5v^{-4.5}$

(i) $\frac{3}{4}x^{-1/4}$ (j) $-\frac{1}{3}t^{-4/3}$ (k) $1.8v^{0.8}$ (l) $-6N^{-7}$

Section 7

(a) $7.5x^{0.5}$ (b) $-12t^{-4}$ (c) $4v^{-1/2}$ (d) $-6n^{-2}$

(e) $v^{-0.5}$ (f) $-15w^{-3.5}$ (g) $52n^{9}$ (h) $\frac{6}{5}t^{5}$

(i) $10t^{-3}$ (j) $6.25p^{1.5}$ (k) $-1.5z^{-1.1}$ (l) $12r^{-5}$

Section 8

(a) $2t+3t^2$ (b) $2+12t^3$ (c) 4 (d) $5-20t$

(e) $13.5t^{3.5}+4t^{-2}$ (f) $2t^{-1/2}+5t^{-3}$ (g) $2x+12x^3$ (h) $-2x^{-2}+2x^{-3}$

(i) $3+\dfrac{y}{2}$

Section 9

(a) $-t^{-2}$ (b) $-4t^{-3}$ (c) $-15x^{-4}$ (d) $-16.8r^{-8}$

(e) $-2.25t^{-3/2}$ (f) $-60t^{-7}$ (g) $-19.6x^{-5}$ (h) $-7.5\theta^{-4}$

(i) $-\pi\theta^{-2}$ (j) $-5\sqrt{2}t^{-6}$ (k) $-\dfrac{7}{2}x^{-2}$ (l) $-7.5r^{-3.5}$

Section 10

(a) $-t^{-2}$ (b) $10t$ (c) $-5t^{-2}-6t^{-3}-3t^{-4}$ (d) $-14x^{-3}-6x^{-2}$

(e) $-\dfrac{3}{4}x^{-2}$ (f) $-\dfrac{7}{3}t^{-2}$ (g) $-4t^{-3}-5t^{-2}+2$ (h) $-\dfrac{32}{3}z^{-5}-4z^{-3}$

(i) $\dfrac{1}{2}$

Section 11

(a) $12(2t+3)^5$ (b) $14(5+2t)^6$ (c) $(0.4t+1)^{1.5}$ (d) $10\pi(\pi x+2.4)^9$

(e) $-10(5t-3)^{-3}$ (f) $18(3-6t)^{-4}$ (g) $2(4x-1)^{-1/2}$ (h) $1.5(2+0.5t)^2$

(i) $-2(4x-6)^{-1.5}$

Section 12

(a) $10(3t^2+2t+4)^4(3t+1)$ (b) $-4(4\theta^2-2\theta+1)^{-3}(4\theta-1)$ (c) $2(8x^2+5x)(16x+5)$

(d) $-3(3t+1)^{-2}$ (e) $-20v(5v^2+2)^{-3}$ (f) $2(t^3+t+1)(3t^2+1)$

(g) $-60x^2(5x^3+1)^{-5}$ (h) $8(2t+1)^3$ (i) $6(3y^2-4y)^2(3y-2)$

Section 13

Exercise 1

(a) $-2t^{-2}$ (b) $\frac{1}{2}t^{-1/2}$ (c) $-8t^{-3}$

(d) $1/3$ (e) $-t^{-2}$ (f) $1+3t^{-2}-8t^{-3}$

(g) $20(5t+1)^3$ (h) $-3(3t+1)^{-2}$ (i) $-6(3t+1)^{-2}$

(j) $10t+3$ (k) $-6t^{-4}+4t^3$ (l) $-5(2t+3t^{-1})^{-6}(2-3t^{-2})$

Exercise 2

(a) $\frac{1}{2}x^{-1/2}$ (b) $\frac{1}{2}(x+1)^{-1/2}$ (c) $-3x^{-2}$

(d) $5-x^{-2}$ (e) $10x$ (f) $70x(x^2+1)^4$

(g) $\frac{2}{3}(2x+1)^{-2/3}$ (h) $-7x^{-2}$ (i) $-\frac{10}{3}x^{-3}$

(j) $-8x^{-3}-3x^{-2}$ (k) $\frac{3}{2}x^{-7/4}$ (l) $2(2x^3+3x^2-5x)(6x^2+6x-5)$

Exercise 3

$(a)\quad 3$	$(b)\quad -2.1t^{-4} - 18t^2$	$(c)\quad -2.5(5t-6)^{-1.5}$
$(d)\quad -4t^{-2} + 3$	$(e)\quad 0$	$(f)\quad -4t^{-3} + 3t^{-2} + 5$
$(g)\quad -3(6t-2)^{-1.5}$	$(h)\quad -6t^{-3}$	$(i)\quad 3 + 0.1t^{-1.1}$
$(j)\quad 32t^3 - 1.5t^{-1/2}$	$(k)\quad 6(5t^{-2} + 3t^2)^2(-5t^{-3} + 3t)$	$(l)\quad -\pi$

Acknowledgements

The materials in **Basic Differentiation - A Refresher** were prepared by David Pidcock at the Mathematics Education Centre, Loughborough University, on behalf of **math**centre.

The document was prepared in Latex by Joan de Souza for **math**centre.

Users are invited to send any suggestions for improvement to enquiries@mathcentre.ac.uk.

This edition of **Basic Differentiation - A Refresher** is published by **math**centre in March 2003.

If departments wish to customise and print their own copies, the contents of the book can be supplied in electronic format suitable for printing.

Further details from:

mathcentre
c/o Mathematics Education Centre
Loughborough University
LE11 3TU

email: enquiries@mathcentre.ac.uk
tel: 01509 227465
www: http://www.mathcentre.ac.uk

